Weighting error – a potential source of systematic measurement errors in process analysis

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The Weighting Error, *WE,* is a systematic error which is generated under certain conditions if the mean value of a lot is estimated as simple arithmetic mean. Cases where this can happen are:

- Lots consisting of several packages (sub-lots) of different sizes, each having different average concentrations. In this case it is obvious that the mean concentration of the analyte in the whole lot should be calculated as a *weighted average* by using the sizes of the sub-lots as statistical weights.

- Samples of constant size (volume or weight) are drawn from the target lot. If the density of the material varies in the lot and is *correlated* with the concentration of the analyte, the simple average estimate of the lot mean is *biased*. A correct average value is obtained if the individual results are weighted by using the densities or the masses of the samples as statistical weights in calculating the mean.

- The flow-rate of the moving stream of matter correlates with the concentration of the analyte and constant-volume samples are drawn and analysed, or combined into a composite sample. This procedure ignores the effect of the flow-rate variation and, consequently, the simple mean of the measurements is biased due to weighting error. As compositing of sub-samples is equivalent of averaging also the composite sample is biased. If the samples are cut with a constant speed proportional cross-stream cutter the sample masses can be used as weights in calculating the average because they are proportional to the flow-rate. Similarly, if these individual increments are combined into a composite sample before analysis, the composite sample is unbiased. If it is not possible to use proportional sampling, the flow-rates at the time of sampling can be used as weights, provided that reliable measurements are available. In this case, if composite samples are made, the increments should be drawn when a predetermined volume has passed the sampling device (volume-proportional sampling) to guarantee an unbiased composite sample.

Gy defined the *WE* in [1], but its properties were not further studied. In later publications Gy did not discuss this error further. His justification was that it can be eliminated if a correct cross-stream proportional sampling devices are used for cutting the samples. The mean of the sampling target (lot) should then be estimated as the weighted mean by using the sample masses as the weights in estimating the mean.

There are, however, many industrial sampling targets where proportional sampling is difficult and expensive to realize in practice, if not impossible. Consider, *e.g.* sampling dust emissions into the atmosphere from a stack of a large industrial soda recovery boiler, which might have a diameter up to eight meters. From this kind of sampling targets it is impossible to cut correct cross-sections from the stream. Only point intake samplers are in use today. A critique of traditional approaches to this type of challenging sampling situation can be found in Wagner & Esbensen [2,3].

As mentioned above, in order to eliminate the weighting error composite samples are sometimes collected so that the sampling device is coupled with a flow-meter, and an increment is taken when a predetermined total volume has passed the sampling point. In this approach the sampling interval is varied proportionally to the flow-rate. Some doubts have been presented concerning the effectiveness of this approach and difficulties have been experienced in convincing industrial partners to adopt this approach, especially because it is more difficult to implement than the systematic sampling at fixed time intervals. It is also often claimed that increasing the number of samples decreases the weighting error in estimating the lot average. It is also claimed sometimes that increasing the number of samples will decrease the weighting error. The study presented here shows, however, that this is not the case.

REFERENCES

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